Show

$$\left(p^\mu \partial_\mu + m F^\mu \frac{\partial}{\partial p^\mu} \right) \theta(p^0) \delta(p^2 + m^2) = 0$$

provided that either $F^\mu p_\mu = 0$ or $F^\mu = -\partial^\mu m = 0$

2. Entropy production (Lecture 1)

Starting from the fluid-fleld coupling model

$$\partial_\mu T_{\rm f}^{\mu\nu} + \partial^\nu\phi \frac{\partial V_T(\phi)}{\partial\phi} = \tilde{\eta}\frac{\phi^2}{T}(U\cdot\partial\phi)\partial^\nu\phi$$

show that the entropy current ($S^\mu = sU^\mu$, $s = dp/dT$) satisfies

$$\partial_\mu S^\mu = \tilde{\eta}\left(\frac{\phi}{T}\right)^2 (U\cdot\partial\phi)^2$$

Don't forget that $T^{\mu\nu} = wU^\mu U^\nu + pg^{\mu\nu}$, $w = Ts$, and $p = g_{\rm eff}\frac{\pi^2}{90}T^4 - V_T(\phi)$.

3. Junction conditions (Lecture 2)

Starting from EM conservation across the bubble wall:

$$w_- \gamma_-^2 \tilde{v}_- = w_+ \gamma_+^2 \tilde{v}_+, \quad w_- \gamma_-^2 \tilde{v}_-^2 + p_- = w_+ \gamma_+^2 \tilde{v}_+^2 + p_+$$

show

$$\tilde{v}_+ \tilde{v}_- = \frac{p_+ - p_-}{e_+ - e_-}, \quad \frac{\tilde{v}_+}{\tilde{v}_-} = \frac{e_- + p_+}{e_+ + p_-}$$

and hence

$$\tilde{v}_+ \tilde{v}_- = \frac{1 - (1 - 3\alpha_+) r}{3 - 3(1 + \alpha_+) r}, \frac{\tilde{v}_+}{\tilde{v}_-} = \frac{3 + (1 - 3\alpha_+) r}{1 + 3(1 + \alpha_+) r}$$

where

- $\theta_\pm = \frac{1}{4}(e_\pm - 3p_\pm)$, $\Delta\theta = \theta_+ - \theta_-$
- $\alpha_+ = \frac{4\Delta\theta}{3w_+}$
- $r = w_+/w_-$

Hint: solve for variables $X = \tilde{v}_+ \tilde{v}_-$, $Y = \tilde{v}_+/\tilde{v}_-$.

4. Sound waves (Lecture 2)

The EM tensor for fluid perturbations with z dependence only is

$$T^{tt} = w\gamma^2 - p, \qquad T^{tz} = w\gamma^2 v^z, \qquad T^{zz} = w\gamma^2 (v^z)^2 + p$$

Consider perturbations: $\delta e = e - \bar{e}$, $\delta p = p - \bar{p}$, v^z all $\ll 1$. Assume that the pressure and and temperature are functions of temperature only $p = p(T)$, $e = e(T)$. Show that the equation for small perturbations is

$$\left(\partial_t^2 - c_s^2 \partial_z^2\right) v^z = 0, \qquad \left(\partial_t^2 - c_s^2 \partial_z\right) \delta e = 0,$$

where $c_s^2 = dp/de$.